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1992 J. Phys. A: Math. Gen. 25 L283

(http://iopscience.iop.org/0305-4470/25/6/003)

J. Phys. A: Math. Gen. 25 (1992) L283–L286. Printed in the UK

LETTER TO THE EDITOR

Dynamics in a dilute ferromagnet at the percolation threshold

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Received 21 November 1991

Abstract. The dynamics of the spin autocorrelation function and the relaxation of the magnetization in the Griffiths phase of the two-dimensional bond-diluted Ising model at the percolation threshold are studied using Monte Carlo techniques. The results resolve a previous ambiguity about the decay law.

Recently, the dynamics of random magnetic systems in the Griffiths phase has been studied by clustering arguments and by Monte Carlo simulations. In particular, it has been argued that the dilute two-dimensional Ising ferromagnet shows a peculiar asymptotic temporal evolution of the spin autocorrelation function [1,2]

$$C(t) = \langle S_i(0) S_i(t) \rangle \sim \exp[-A(\ln t)^2] \tag{1}$$

above the percolation threshold, $p \geqslant p_c$, at temperatures $T_c(p) < T < T_c(p=1)$, where $T_c(p)$ is the phase transition temperature, and $(1-p)$ is the concentration of missing bonds or sites; the amplitude A depends on the system parameters, such as T and p.

This theoretical prediction has been compared to results of Monte Carlo simulations [3–5] of the nearest-neighbour bond-diluted Ising model on a square lattice. Its Hamiltonian may be written in the form

$$\mathcal{H} = -\sum_{i,j} J_{ij} S_i S_j \tag{2}$$

where the exchange couplings J_{ij} are chosen from the distribution

$$P(J_{ij} = (1-p)\delta(J_{ij} - J') + p\delta(J_{ij} - J)\,. \tag{3}$$

Simulations have been performed for the case $J' = 0$ and $J = 1$ at the percolation threshold, $p = p_c = \frac{1}{2}$ [3,4] and slightly above p_c [5].

At the percolation threshold, the Monte Carlo data of both studies [3,4] have been found to satisfy rather well a stretched exponential form, even at quite small times

$$C(t) \sim \exp[-(t/\tau)^{\beta}] \tag{4}$$

up to the latest times monitored in the simulations. The exponent β turned out to increase with temperature.

Conspiciously, the results were claimed to be also consistent with the anomously slow decay predicted by (1). At the latest times of those simulations, the Monte Carlo data plotted as $\ln(-\ln C)$ against $\ln(\ln t)$ were observed to approach a straight line of slope 2, see figure 2 of [3], and figure 11 of [4].

In this letter, we shall address this ambiguity by presenting results of simulations on the bond-diluted Ising model, equations (2) and (3), with $J' = 0$ and $J = 1$ at the percolation threshold, $p = \frac{1}{2}$, in the Griffiths phase $0 \leqslant T \leqslant T_c(1) = 2/\ln(\sqrt{2} + 1) = 2.269\ldots$ We used two approaches to study the dynamics of the model. Firstly, we monitored the spin autocorrelation function, $C(t)$, and, secondly, we recorded the relaxation of the magnetization, $M(t)$, from the ordered ground state, $M(t = 0) = 1$, to its equilibrium value, $M(t = \infty) = 0$. Following the arguments by Colborne and Bray [4], the non-equilibrium decay of the magnetization is expected to be described by the same asymptotic time dependence as the equilibrium autocorrelation function.

To solve the problem, larger system sizes, lower temperatures and longer time scales (up to a factor of about three) have been studied. To avoid possible artefacts, the Monte Carlo algorithm with random updating has been employed, in contrast to the systematic sublattice updating, allowing for efficient vectorization [3, 4].

Specifically, systems of sizes 16^2, 32^2, and 256^2 with full periodic boundary conditions were studied, with the number of samples ranging from several tens of thousands to a few hundred. Computations were done on the entire percolation lattice and not only on the backbone. Statistical error analyses were performed on the number of samples in the standard way. The temperatures advanced from $T = 1.0$ to $T = 1.5$ in steps of 0.1.

The computations for $C(t)$ were carried out on a special purpose computer at the Landau Institute [6]; $M(t)$ was determined on a scalar IBM computer in Jülich.

Our main findings are summarized in figures 1 to 4. As depicted in figure 1, the autocorrelation function can be fitted rather well by a stretched exponential form, equation (4), with a temperature-dependent exponent β, ranging from $\beta \approx 0.32$ at $T = 1.0$ up to $\beta \approx 0.52$ at $T = 1.5$. The exponents are 'effective' ones, disregarding the data at early times and keeping in mind the slight upward curvature in the data. In the temperature range of the previous simulations, our estimated values for $\beta(T)$ are close to those obtained before using systematic sublattice updating [4].

By plotting $\ln(-\ln C)$ against $\ln \ln t$, see figure 2, one observes at early times an increase of the slope towards two, in accordance with equation (1) and the previous simulations. However, by extending the time scale beyond that of the previous simulations, the slope continues to increase to values clearly larger than two. Therefore, if equation (1) describes the asymptotic behaviour correctly, one seems to be still far from that behaviour even at the latest time studied here. This fact is rather surprising, because equation (1) had been argued to be valid even in the time range of the previous simulations [1, 3].

The relaxation of the magnetization, $M(t)$, shows the same characteristics as the dynamics of $C(t)$, as depicted in figures 3 and 4. Again, the stretched exponential form describes the Monte Carlo data rather well. The estimated values for $\beta(T)$ are appreciably higher than those obtained from the autocorrelation function, ranging from $\beta \approx 0.40$ at $T = 1.0$ up to $\beta \approx 0.59$ at $T = 1.4$. Again, the exponents have to be interpreted as 'effective' ones. There are only minor changes in β due to the random updating as compared to the systematic updating algorithm [4]. As has been pointed out by Colborne and Bray [4], the differences in the values of β for $C(t)$ and $M(t)$ may pose severe problems for taking the stretched exponential form seriously

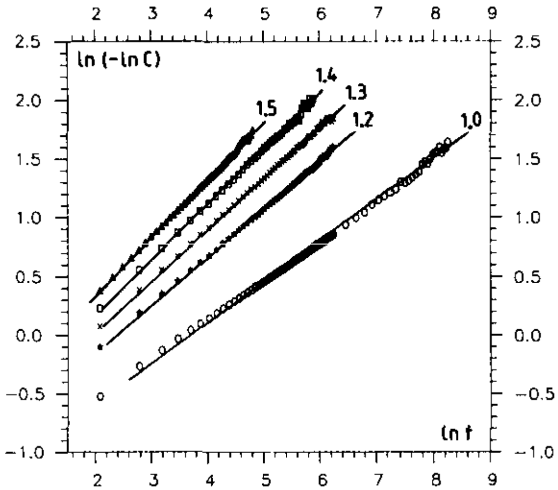

Figure 1. $\ln(-\ln C)$ against $\ln t$, for the temperatures $T = 1.0$, 1.2, 1.3, 1.4, and 1.5, with size $L = 256$. The full lines correspond to stretched exponential forms, equation (4).

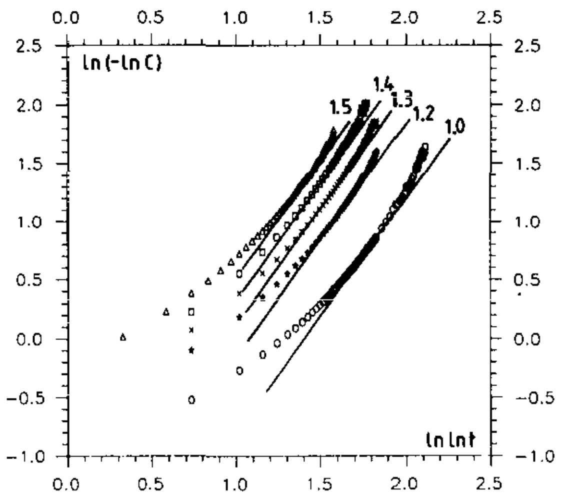

Figure 2. $\ln(-\ln C)$ against $\ln(\ln t)$, for the temperatures $T = 1.0$, 1.2, 1.3, 1.4, and 1.5, with size $L = 256$. The full lines, included as guides to the eye, have slope 2, see equation (1).

as a candidate asymptotic form. On the other hand, as shown in figure 4, there is, so far, no evidence that the decay law of (1) is approached. On the contrary, at the latest times a strong upturn from the straight line of slope 2 occurs by replotting the data in the form $\ln(-\ln M)$ against $\ln(\ln t)$, even more drastically than in the case of the autocorrelation function, figure 2. The upturn occurred for all sizes studied, $16 \leqslant L \leqslant 256$, in accordance with supposedly minor finite size effects.

In summary, the new Monte Carlo data, describing the dynamics of the autocorrelation function and the relaxation of the magnetization, can be fitted quite well by the stretched exponential form. There is no evidence for an even slower temporal evolution on the time scales accessible to the simulations presented here, resolving the previous ambiguity about the decay law. (An intriguing aspect remains, in that the streched exponential behaviour violates a supposedly rigorous bound of the form (1) on the ultimate asymptotic decay law [1,2]; a similar remark applies to the work by Ogielski on the simulation of spin glasses [7], which also did not encounter this

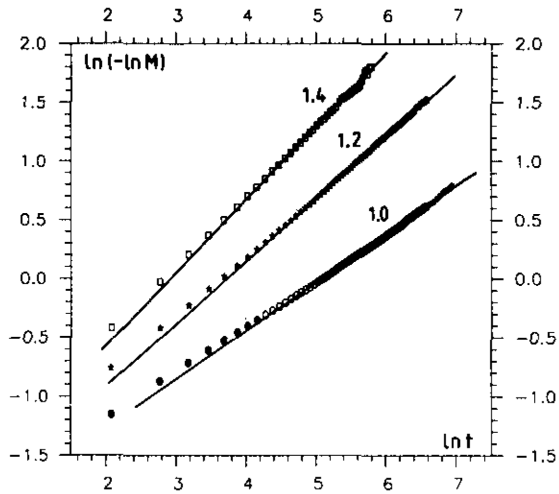

Figure 3. $\ln(-\ln M)$ against $\ln t$, for the temperatures $T = 1.0$, 1.2, and 1.4, with size $L = 256$ (data for $T = 1.0$ with $L = 32$ are denoted by crosses). The full lines correspond to stretched exponential forms.

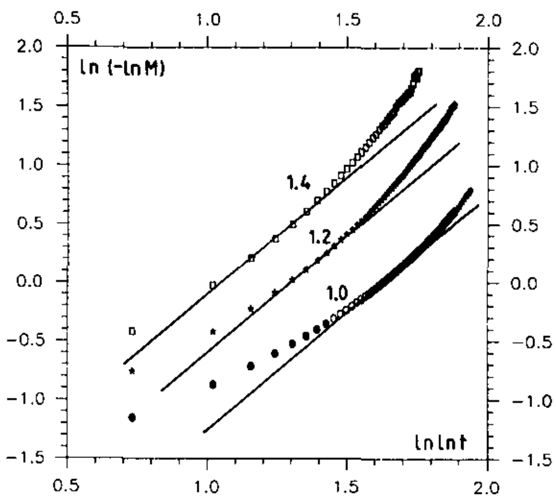

Figure 4. $\ln(-\ln M)$ against $\ln(\ln t)$, for the temperatures $T = 1.0$, 1.2, and 1.4, with size $L = 256$ (data for $T = 1.0$ with $L = 32$ are denoted by crosses). The full lines, included as guides to the eye, have slope 2.

constraint.) Currently, we are studying the effects caused by allowing for interactions between clusters in taking $J' > 0$ (for critical properties, see [8]) and by moving away from the percolation threshold, $p \neq p_c$.

Useful discussions with Vl S Dotsenko and D P Landau are gratefully acknowledged.

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